

Appendix I: A GRAPHICAL TOUR OF THE BOOK

For those nonliterary folks who like to see things in graphs, rather than words, and those only.

NONLINEARITY AND LESS IS MORE (& PROCRUSTEAN BED)

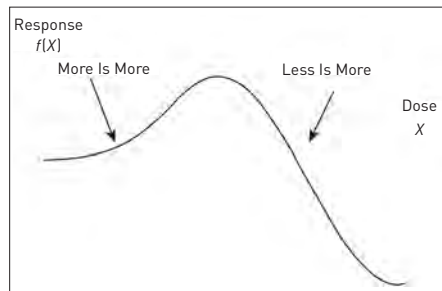


FIGURE 19. This graph explains both the nonlinear response and the “less is more” idea. As the dose increases beyond a certain point, benefits reverse. We saw that everything nonlinear is either convex, concave, or, as in this graph, mixed. Also shows how under nonlinearities, reductions fail: the Procrustean bed of words “good for you” or “bad” is severely distorting.

Also shows why tinkering-derived heuristics matter because they don’t take you into the danger zone—words and narratives do. Note how the “more is more” zone is convex, meaning accelerated initial benefits. (In Levantine Arabic, the zone beyond the saturation has a name: **كترتا مثل قلنا** “more of it is like less of it.”)

Finally, it shows why competitive “sophistication” (rather, complication masked as sophistication) is harmful, as compared to the practitioner’s craving for optimal simplicity.

Fragility Transfer Theorem:

Note that by the Fragility Transfer Theorem,

CONVEX EXPOSURE [OVER SOME RANGE] ↔ LIKES VOLATILITY [UP TO SOME POINT]

(volatility and other members of the disorder cluster), and

CONCAVE EXPOSURE ↔ DISLIKES VOLATILITY

MAPPING OF FRAGILITIES

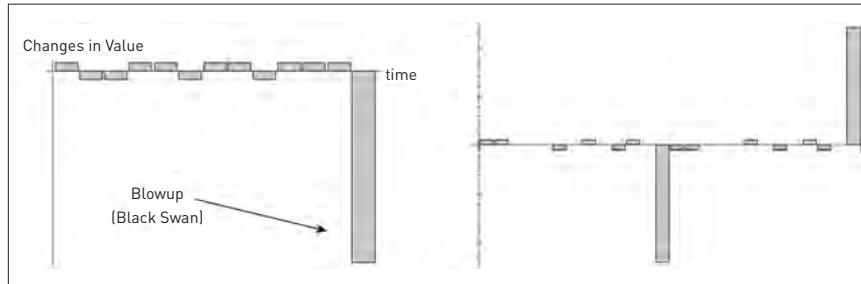
In Time Series Space

FIGURE 20. *Fragile variations through time, two types of fragilities. A representative series. The horizontal axis shows time, the vertical one shows variations. This can apply to anything: a health indicator, changes in wealth, your happiness, etc. We can see small (or no) benefits and variations most of the time and occasional large adverse outcomes. Uncertainty can hit in a rather hard way. Notice that the loss can occur at any time and exceed the previous cumulative gains. Type 2 (left) and Type 1 (right) differ in that Type 2 does not experience large positive effects from uncertainty while Type 1 does.*

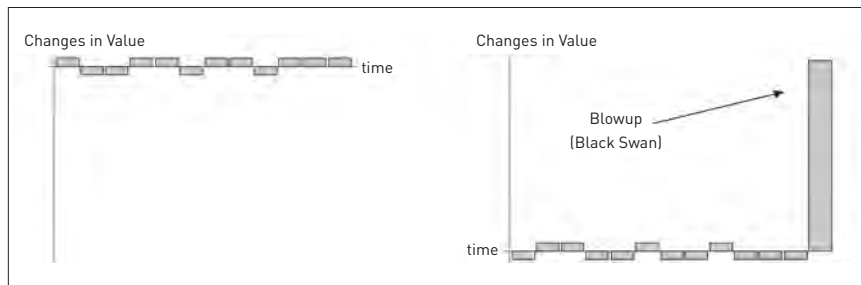
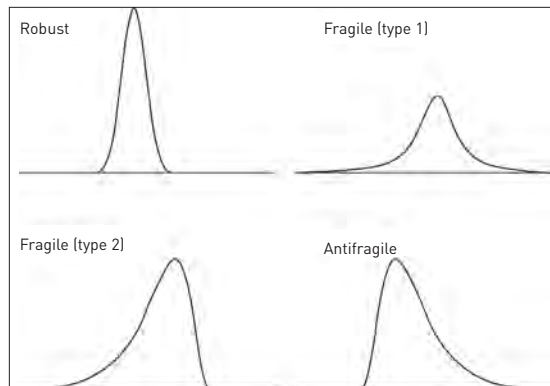


FIGURE 21. *The Just Robust (but not antifragile) (on the left): It experiences small or no variations through time. Never large ones. The Antifragile system (right): Uncertainty benefits a lot more than it hurts—the exact opposite of the first graph in Figure 20.*

Seen in Probabilities

FIGURE 22. *The horizontal axis represents outcomes, the vertical their probability (i.e., their frequency). The Robust: Small positive and negative outcomes. The Fragile (Type 1, very rare): Can deliver both large negative and large positive outcomes. Why is it rare? Symmetry is very, very rare empirically yet all statistical distributions tend to simplify by using it. The Fragile (Type 2): We see large improbable downside (often hidden and ignored), small upside. There is a possibility of a se-*



vere unfavorable outcome (left), much more than a hugely favorable one, as the left side is thicker than the right one. **The Antifragile:** Large upside, small downside. Large favorable outcomes are possible, large unfavorable ones less so (if not impossible). The right “tail,” for favorable outcomes, is larger than the left one.

TABLE 9 • THE FOUR DIFFERENT CLASSES OF PAYOFFS

Left Tail of the Distribution	Right Tail of the Distribution	Condition
Thin	Thick	Antifragile
Thick	Thick	Fragile (Type 1) [Very Rare]
Thick	Thin	Fragile (Type 2)
Thin	Thin	Robust

Fragility has a left tail and, what is crucial, is therefore sensitive to perturbations of the left side of the probability distribution.

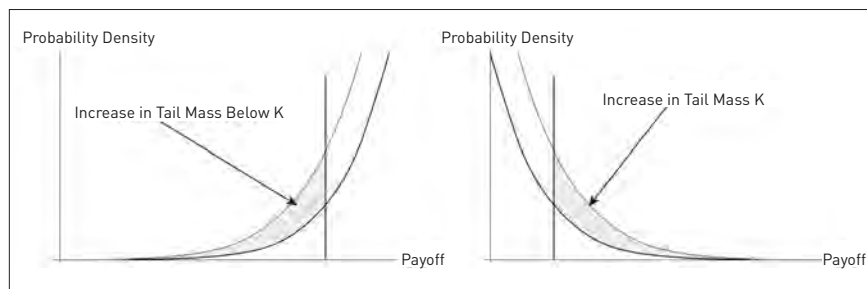


FIGURE 23. Definition of Fragility (left graph): Fragility is the shaded area, the increase in the mass in left tail below a certain level K of the target variable in response to any change in parameter of the source variable—mostly the “volatility” or something a bit more tuned. We subsume all these changes in s^- , about which later in the notes section (where I managed to hide equations).

For a definition of antifragility (right graph), which is not exactly symmetric, the

same mirror image for right tail plus robustness in left tail. The parameter perturbed is s^* .

It is key that while we may not be able to specify the probability distribution with any precision, we can probe the response through heuristics thanks to the “transfer theorem” in Taleb and Douady (2012). In other words, we do not need to understand the future probability of events, but we can figure out the fragility to these events.

BARBELL TRANSFORMATION IN TIME SERIES

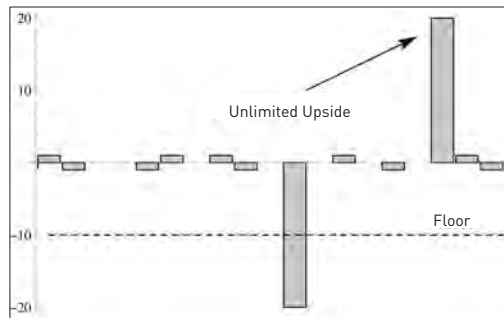
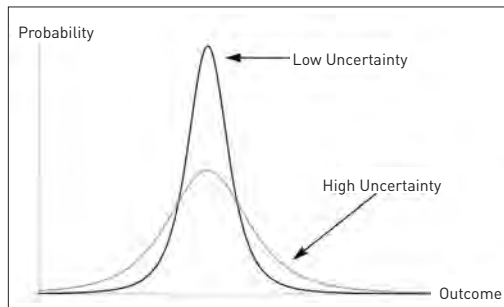


FIGURE 24. Barbell seen in time series space. Flooring payoffs while keeping upside.

BARBELLS (CONVEX TRANSFORMATIONS) AND THEIR PROPERTIES IN PROBABILITY SPACE



A graphical expression of the barbell idea.

FIGURE 25. Case 1, the Symmetric Case. Injecting uncertainty into the system makes us move from one bell shape—the first, with narrow possible spate of outcomes—to the second, a lower peak but more spread out. So it causes an increase of both positive and negative surprises, both positive and negative Black Swans.

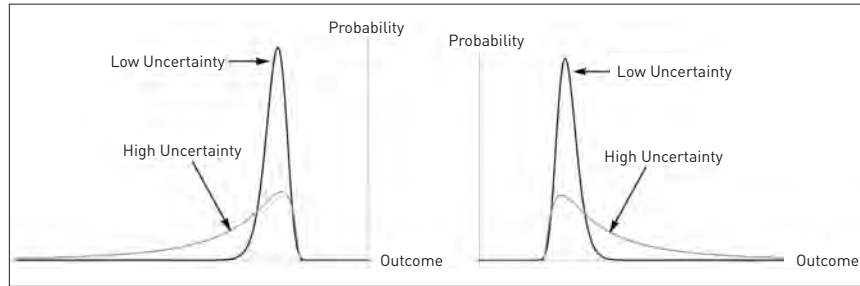


FIGURE 26. Case 2 (left): *Fragile*. Limited gains, larger losses. Increasing uncertainty in the system causes an augmentation of mostly (sometimes only) negative outcomes, just negative Black Swans. Case 3 (right): *Antifragile*. Increasing randomness and uncertainty in the system raises the probability of very favorable outcomes, and accordingly expand the expected payoff. It shows how discovery is, mathematically, exactly like an anti-airplane delay.

TECHNICAL VERSION OF FAT TONY'S "NOT THE SAME 'TING,'" OR THE CONFLATION OF EVENTS AND EXPOSURE TO EVENTS

This note will also explain a "convex transformation."

$f(x)$ is *exposure* to the variable x . $f(x)$ can equivalently be called "payoff from x ," "exposure to x ," even "utility of payoff from x " where we introduce in f a utility function. x can be anything.

Example: x is the intensity of an earthquake on some scale in some specific area, $f(x)$ is the number of persons dying from it. We can easily see that $f(x)$ can be made more predictable than x (if we force people to stay away from a specific area or build to some standards, etc.).

Example: x is the number of meters of my fall to the ground when someone pushes me from height x , $f(x)$ is a measure of my physical condition from the effect of the fall. Clearly I cannot predict x (who will push me, rather $f(x)$).

Example: x is the number of cars in NYC at noon tomorrow, $f(x)$ is travel time from point A to point B for a certain agent. $f(x)$ can be made more predictable than x (take the subway, or, even better, walk).

Some people talk about $f(x)$ thinking they are talking about x . This is the problem of the **conflation of event and exposure**. This error present in Aristotle is virtually ubiquitous in the philosophy of probability (say, Hacking).

One can become antifragile to x without understanding x , through convexity of $f(x)$.

The answer to the question "what do you do in a world you don't understand?" is, simply, work on the undesirable states of $f(x)$.

It is often easier to modify $f(x)$ than to get better knowledge of x . (In other words, robustification rather than forecasting Black Swans.)

Example: If I buy an insurance on the market, here x , dropping more than 20 percent, $f(x)$ will be independent of the part of the probability distribution of x that is below 20 percent and impervious to changes in its scale parameter. (This is an example of a barbell.)

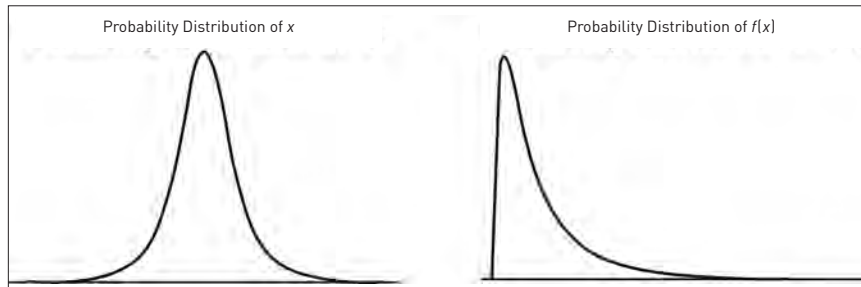


FIGURE 27. Convex Transformation ($f(x)$ is a convex function of x). The difference between x and exposure to x . There is no downside risk in the second graph. The key is to modify $f(x)$ in order to make knowledge of the properties of x on the left side of the distribution as irrelevant as possible. This operation is called convex transformation, nicknamed “barbell” here.

Green lumber fallacy: When one confuses $f(x)$ for another function $g(x)$, one that has different nonlinearities.

More technically: If one is antifragile to x , then the variance (or volatility, or other measures of variation) of x benefit $f(x)$, since distributions that are skewed have their mean depend on the variance and when skewed right, their expectation increases with variance (the lognormal, for instance, has for mean a term that includes $+\frac{1}{2}\sigma^2$).

Further, the probability distribution of $f(x)$ is markedly different from that of x , particularly in the presence of nonlinearities.

When $f(x)$ is convex (concave) monotonically, $f(x)$ is right (left) skewed.

When $f(x)$ is increasing and convex on the left then concave to the right, the probability distribution of $f(x)$ is thinner-tailed than that of x . For instance, in Kahneman-Tversky’s prospect theory, the so-called utility of changes in wealth is more “robust” than that of wealth.

Why payoff matters more than probability (technical): Where $p(x)$ is the density, the expectation, that is $\int f(x)p(x)dx$, will depend increasingly on f rather than p , and the more nonlinear f , the more it will depend on f rather than p .

THE FOURTH QUADRANT (TALEB, 2009)

The idea is that tail events are not computable (in fat-tailed domains), but we can assess our exposure to the problem. Assume $f(x)$ is an increasing function, Table 10 connects the idea to the notion of the Fourth Quadrant.

TABLE 10

	<i>THIN-TAILED DISTRIBUTION FOR X</i>	<i>FAT-TAILED DISTRIBUTION FOR X</i>
$f(x)$ "mitigating" by clipping extreme outcomes, i.e., convex-concave	Very robust outcome	Quite robust outcome
$f(x)$ concave-convex, exacerbates remote outcomes	Robust outcome (sort of)	FOURTH QUADRANT Fragile (if $f(x)$ is concave) or antifragile

LOCAL AND GLOBAL CONVEXITIES (TECHNICAL)

Nothing is open-ended in nature—death is a maximum outcome for a unit. So things end up convex on one end, concave on the other.

In fact, there is maximum harm at some point in things biological. Let us revisit the concave figure of the stone and pebbles in Chapter 18: by widening the range we see that boundedness of harm brings convexities somewhere. Concavity was dominant, but local. Figure 28 looks at the continuation of the story of the stone and pebbles.

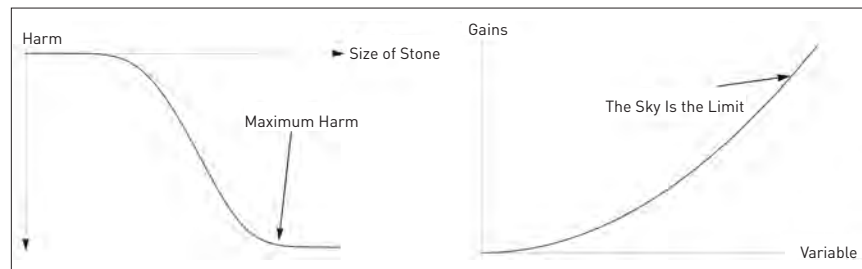


FIGURE 28. The left graph shows a broader range in the story of the stone and pebbles in Chapter 18. At some point, the concave turns convex as we hit maximum harm. The right graph shows strong antifragility, with no known upper limit (leading to Extremistan). These payoffs are only available in economic variables, say, sales of books, or matters unbounded or near-unbounded. I am unable to find such an effect in nature.

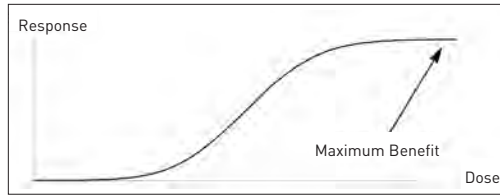


FIGURE 29. *Weak Antifragility (Mediocristan), with bounded maximum. Typical in nature.*

FREAK NONLINEARITIES (VERY TECHNICAL)

The next two types of nonlinearities are almost never seen outside of economic variables; they are particularly limited to those caused by derivatives.

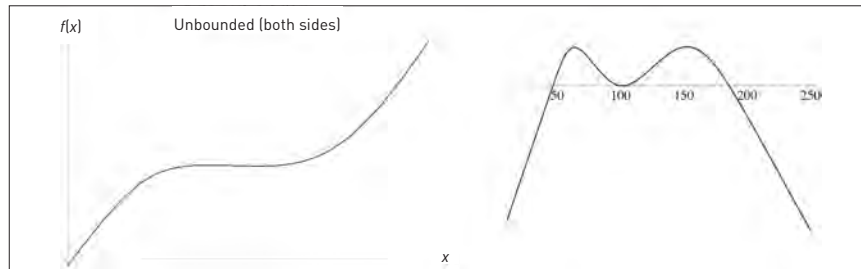


FIGURE 30. *The left graph shows a convex-concave increasing function, the opposite of the bounded dose-response functions we see in nature. It leads to Type 2, Fragile (very, very fat tails). The right graph shows the most dangerous of all: pseudoconvexity. Local antifragility, global fragility.*

MEDICAL NONLINEARITIES AND THEIR PROBABILITY CORRESPONDENCE (CHAPTERS 21 & 22)

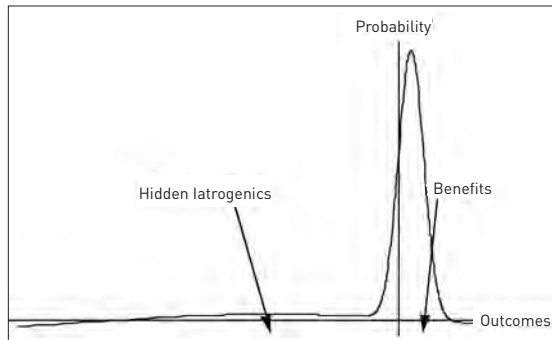


FIGURE 31. *Medical Iatrogenics: Case of small benefits and large Black Swan-style losses seen in probability space. Iatrogenics occurs when we have small identifiable gains (say, avoidance of small discomfort or a minor infection) and exposure to Black Swans with delayed invisible large side effects (say, death). These*

concave benefits from medicine are just like selling a financial option (plenty of risk) against small tiny immediate gains while claiming “evidence of no harm.”

In short, for a healthy person, there is a small probability of disastrous outcomes (discounted because unseen and not taken into account), and a high probability of mild benefits.

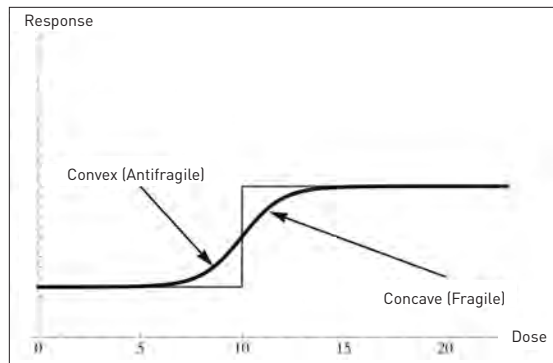


FIGURE 32. Nonlinearities in biology. The shape convex-concave necessarily flows from anything increasing (monotone, i.e., never decreasing) and bounded, with maximum and minimum values, i.e., does not reach infinity from either side. At low levels, the dose response is convex (gradually more and more effective).

Additional doses tend to become gradually ineffective or start hurting. The same can apply to anything consumed in too much regularity. This type of graph necessarily applies to any situation bounded on both sides, with a known minimum and maximum (saturation), which includes happiness.

For instance, if one considers that there exists a maximum level of happiness and unhappiness, then the general shape of this curve with convexity on the left and concavity on the right has to hold for happiness (replace “dose” with “wealth” and “response” with “happiness”). Kahneman-Tversky prospect theory models a similar shape for “utility” of changes in wealth, which they discovered empirically.

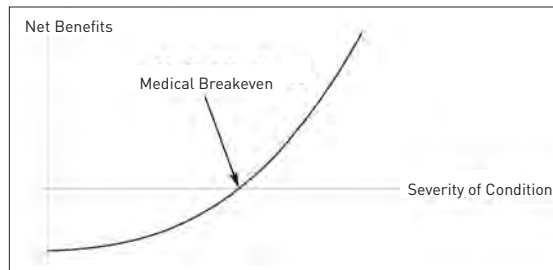


FIGURE 33. Recall the hypertension example. On the vertical axis, we have the benefits of a treatment, on the horizontal, the severity of the condition. The arrow points at the level where probabilistic gains match probabilistic harm. Iatrogenics disappears nonlinearly as a function of the severity of the condition.

This implies that when the patient is very ill, the distribution shifts to antifragile (thicker right tail), with large benefits from the treatment over possible iatrogenics, little to lose.

Note that if you increase the treatment you hit concavity from maximum benefits, a zone not covered in the graph—seen more broadly, it would look like the preceding graph.

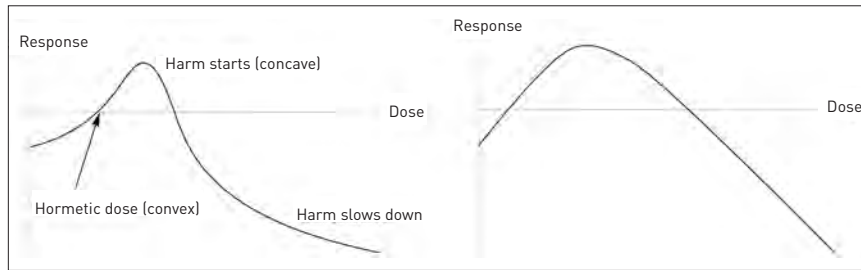


FIGURE 34. The left graph shows hormesis for an organism (similar to Figure 19): we can see a stage of benefits as the dose increases (initially convex) slowing down into a phase of harm as we increase the dose a bit further (initially concave); then we see things flattening out at the level of maximum harm (beyond a certain point, the organism is dead so there is such a thing as a bounded and known worst case scenario in biology). To the right, a wrong graph of hormesis in medical textbooks showing initial concavity, with a beginning that looks linear or slightly concave.

THE INVERSE TURKEY PROBLEM

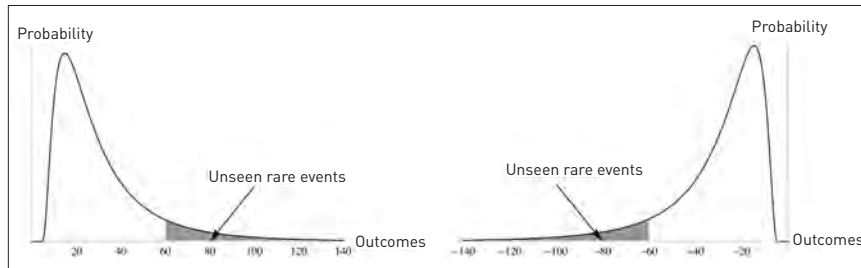


FIGURE 35. Antifragile, Inverse Turkey Problem: The unseen rare event is positive. When you look at a positively skewed (antifragile) time series and make inferences about the unseen, you miss the good stuff and underestimate the benefits (the Pisano, 2006a, 2006b, mistake). On the right, the other Harvard problem, that of Froot (2001). The filled area corresponds to what we do not tend to see in small samples, from insufficiency of points. Interestingly the shaded area increases with model error. The more technical sections call this zone ω_B (turkey) and ω_C (inverse turkey).

DIFFERENCE BETWEEN POINT ESTIMATES AND DISTRIBUTIONS

Let us apply this analysis to how planners make the mistakes they make, and why deficits tend to be worse than planned:

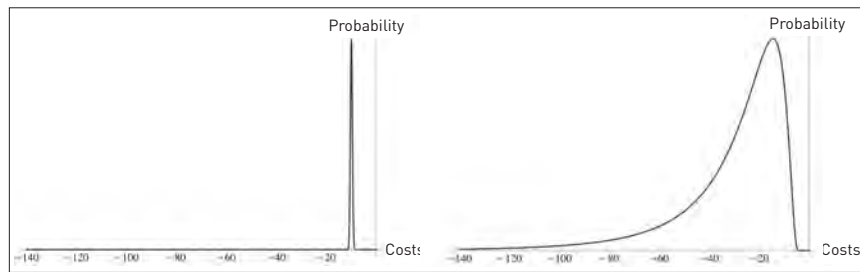


FIGURE 36. *The gap between predictions and reality: probability distribution of outcomes from costs of projects in the minds of planners (left) and in reality (right). In the first graph they assume that the costs will be both low and quite certain. The graph on the right shows outcomes to be both worse and more spread out, particularly with higher possibility of unfavorable outcomes. Note the fragility increase owing to the swelling left tail.*

This misunderstanding of the effect of uncertainty applies to government deficits, plans that have IT components, travel time (to a lesser degree), and many more. We will use the same graph to show model error from underestimating fragility by assuming that a parameter is constant when it is random. This is what plagues bureaucrat-driven economics (next discussion).