Bitcoin, Currencies, and Fragility: Supplementary Discussions

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Rational Expectations

Discretely seen, a price is expected cash flow received at the end of the next period $t+1$ plus expected price at period $t+1$. So let P_t , C_t, and I_t be the price, cash flow (payout to investor) and information, respectively, at period t, with r_d the discount rate. Without any loss, we simplify by assuming C_i and r_d are not stochastic. We note that "cash flow" to investor includes any payout, not just dividend, so C_t includes the liquidation value.

$$P_t = \frac{1}{1+r_d}\left(C_{t+1} + \underbrace{\mathbb{E}(P_{t+1}|_{I_t})}_{\frac{1}{1+r_d}\left(C_{t+2}+\mathbb{E}\left(\underbrace{\mathbb{E}(P_{t+2}|_{I_{t+1}})|_{I_t}}_{\dots}\right)\right)} \right), \tag{1}$$

By the law of iterated expectations,

$$\mathbb{E}\left(\mathbb{E}(P_{t+2}|_{I_{t+1}})|_{I_t}\right) = \mathbb{E}(P_{t+2}|_{I_t}).$$

Allora, noting that, at the present, seen from period t, $\mathbb{E}(P_{t+1}|_{I_t})$ is written as $\mathbb{E}(P_{t+1})$.

$$P_t = \lim_{n\to\infty}\left(\underbrace{\sum_{i=1}^{n}\left(\frac{1}{1+r_d}\right)^i C_{t+i}}_{=0 \text{ for bitcoin}} + \left(\frac{1}{1+r_d}\right)^n \mathbb{E}(P_{t+n}) \right), \tag{2}$$

We notice that the second term vanishes under the smallest positive discount rate. In the standard rational bubble model [1] P (actually, its equivalent, the component that doesn't translate into future cash flow) needs to grow around r_d *forever*. Cases of P growing faster than r_d are never considered as the price becomes explosive (intuitively, given that we are dealing with infinities, it would exceed the value of the economy) [2].

As we increase n, additional cash goes into C_{t+n}; in principle, for $n\to\infty$ it must be all cash outside of bubbles.

Earning-free Assets with Absorbing Barrier

Now, bitcoin is all in the second term, with a hitch: there is an absorbing barrier — should there be an interruption of the ledger updating process, some loss of interest in it, a technological replacement, its value is gone forever. As we insist, bitcoin requires distributed attention.

We define the stopping time as $\tau \triangleq \inf\{n>0; P_{t+n}=0\}$, with $P_{>\tau}=0$.

> **Comment 1: Failure rate**
> *Critically the probability of hitting the barrier does not need to come from price dynamics, but from any failure rate — the only assumption here is a failure rate >0.*

So we impose a layer on top of the dynamics.

$$\mathbb{E}(P_{t+n}) = \mathbb{E}(P_{t+n}|_{t+n<\tau})\,\mathbb{P}(t+n<\tau) + \underbrace{\mathbb{E}(P_{t+n}|_{t+n\geq\tau})}_{=0}\,\mathbb{P}(t+n\geq\tau) \tag{3}$$

Let π be the probability of being absorbed over a single period. Rewriting Eq. 1 with no cash flow, i.e. $C_{t+i}=0\ \forall i$, and eliminating cases for which the expectation is infinite:

$$P_t = \frac{1}{1+r_d}\left((1-\pi)\underbrace{\mathbb{E}(P_{t+1}|_{I_t}|_{(t+1)<\tau}))}_{\frac{1}{1+r_d}\left((1-\pi)\ \underbrace{\mathbb{E}(P_{t+2}|_{I_t}|_{(t+2)<\tau})}_{\dots}\right)} \right). \tag{4}$$

We therefore have

$$P_t = \lim_{n\to\infty}\left(\frac{1-\pi}{1+r_d}\right)^n \mathbb{E}(P_{t+n}|_{(t+n)<\tau}) = 0 \tag{5}$$

For the price to be positive now, P_t must grow *forever*, exactly at a gigantic exponential scale, $e^{n(r+\pi)}$, without remission, and with total certainty.

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Comment 2: The problem of P_∞

The argument that P can grow faster than $e^{n(r+\pi)}$ for a while and accumulate valuation is insufficient: once it stops growing, by backward induction, future absorption makes P_t valued at 0. Remember that we are dealing with infinities.

Furthermore variable mortality rates makes the needed growth vastly in excess of both rates r_d and π. Let π be stochastic with realizations $\pi(1+a)$ and $\pi(1-a)$ — two Diracs at the mean deviation of π. Then the required growth rate must be $e^{n(r+\pi+\sigma)}$, where $\sigma = \frac{\log(\cosh(\pi a n))}{n}$, an additional convexity term $\sigma \approx a\pi$.

References

[1] O. J. Blanchard and M. W. Watson, "Bubbles, rational expectations and financial markets," *NBER working paper*, no. w0945, 1982.

[2] M. K. Brunnermeier, "Bubbles," in *Banking Crises*. Springer, 2016, pp. 28–36.