Recovering Natural Probabilities From Option Prices: A Comment

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***Abstract*—The recovery theorem require Markov conditions between transitional states, but data shows that the path, particularly the position with respect to recent minimum matters for states, significantly enough for equal movement Δp between a price p and $p - \Delta p$ depend severely on whether the latter price is above the recent minimum .**

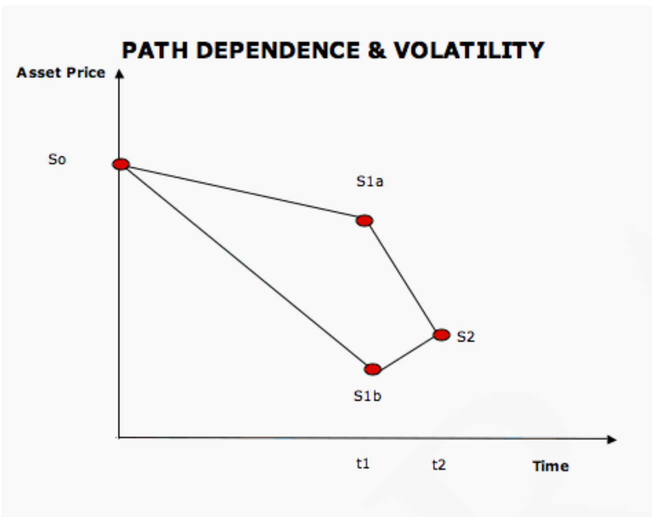

Figure 1. The forward kernel at S2 depends on the path. Implied vol at S2 via S1b is much lower than implied vol at S2 via S1a.

I. Problems With the Revovery theorem

- First, the recovery theorem hinges on the pricing kernel being Markov, and path independent, that is, at times t, the $t+1$, θ, the state of the system (which we can simplify by focusing on σ, the "volatility" at which the distribution is priced, should not depend on the path.
- Second, more devastating for the argument, the recovery theorem is similar to variance swaps with "capped" variance swaps, nice try but not the real McCoy. Alas, probabilities deliver a single segment of the distribution, not full moments. Option prices are not determined by just probabilities but by a combination of probability and payoff, hence the extreme part of the distribution (in fat-tailed domains) may represent a disproportionate segment of the valuation of the *volatility smile*. The current derivations by Ross (2013), and in the continuous time version (Carr et al., 2012) are bounded.

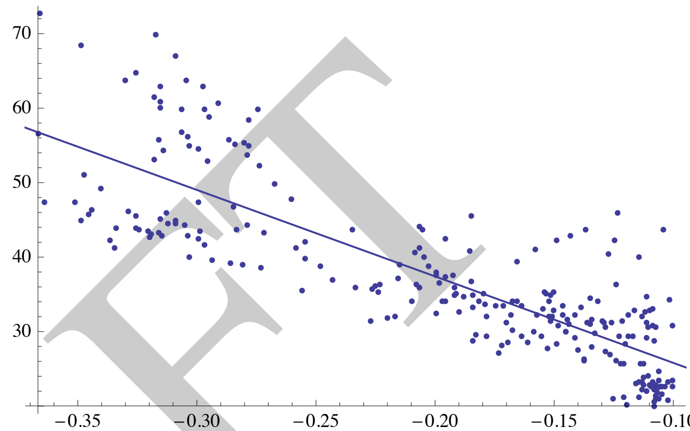

Figure 2. Above the recent minimum, the slope is -116.

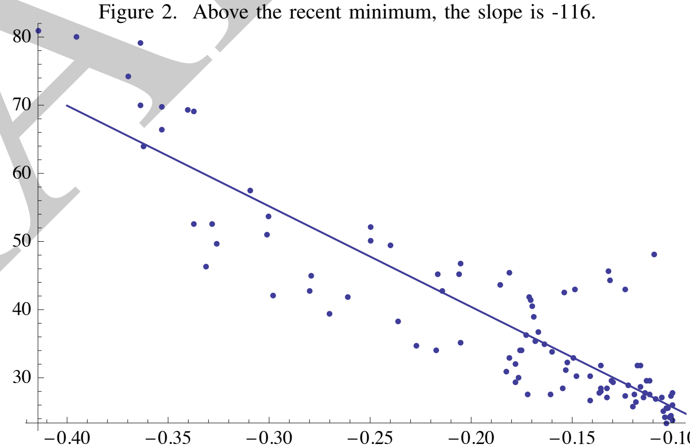

Figure 3. At the new recent lows the slope is -147.

A. Path dependence

We show strong evidence that the transition Arrow-Debreu state prices are not Markov. In other words, at times t, the probability distribution starting from State $S_{t,z}$ varies on whether we got to that state "from above" or "from below". The effect is that we cannot use the marginal rate of substitution (as ratio of utilities of states) without conditioning it on past full sample path.

In other words we are able to do "sum over histories" with impunity for "arbitrage" probabilities (knowing these are pseudoprobabilities) but it is not possible to extract utility and build a "natural probability distribution" because utility is

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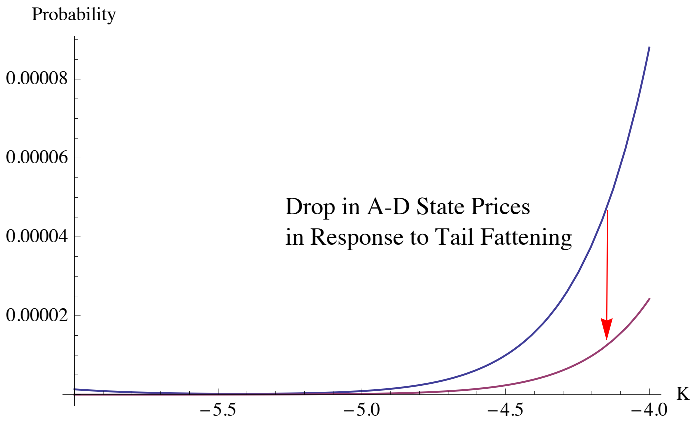

Figure 4. "Pseudoprobabilities" and Tail Events.

concave (or nonlinear), without specification of the distribution of the minimum, etc.

The Regression The regression line of V(t) and has a slope of -147 while the second one (above lows) has one of -116

$\{10.8776, -147.651\}$,

	DF	SS	MS	F-Statistic	P-Value
x	1	15664.8	15664.8	391.943	5.07638×10^{-36}
Error	98	3916.77	39.967		
Total	99	19581.6			

{14.1951,-116.078},

	DF	SS	MS	F-Statistic	P-Value
x	1	21938.5	21938.5	549.093	8.11559×10^{-65}
Error	248	9908.63	39.9542		
Total	249	31847.2			

References

Ross, S. (2013). The recovery theorem. *The Journal of Finance*.

Carr, P., Yu, J. (2012). Risk, return, and Ross recovery. textitJournal of Derivatives, 20(1), 38.

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